2.6 — Statistical Inference

ECON 480 • Econometrics • Fall 2021

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Outline

Why Uncertainty Matters

Confidence Intervals

Confidence Intervals Using the infer Package

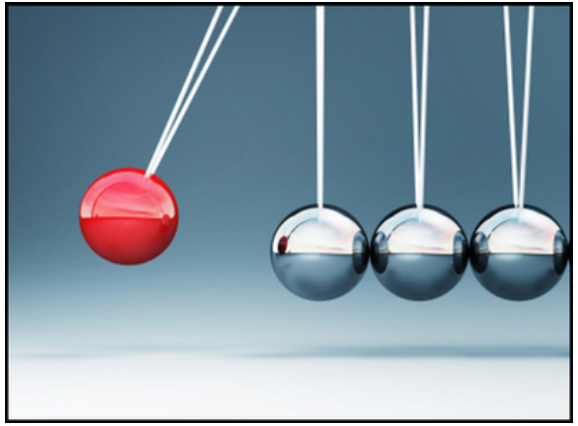
Why Uncertainty Matters

Recall: The Two Big Problems with Data

- We use econometrics to **identify** causal relationships and make **inferences** about them

1. Problem for **identification**: **endogeneity**
   - X is **exogenous** if $cor(x, u) = 0$
   - X is **endogenous** if $cor(x, u) \neq 0$

2. Problem for **inference**: **randomness**
   - Data is random due to **natural sampling variation**
   - Taking one sample of a population will yield slightly different information than another sample of the same population



Distributions of the OLS Estimators

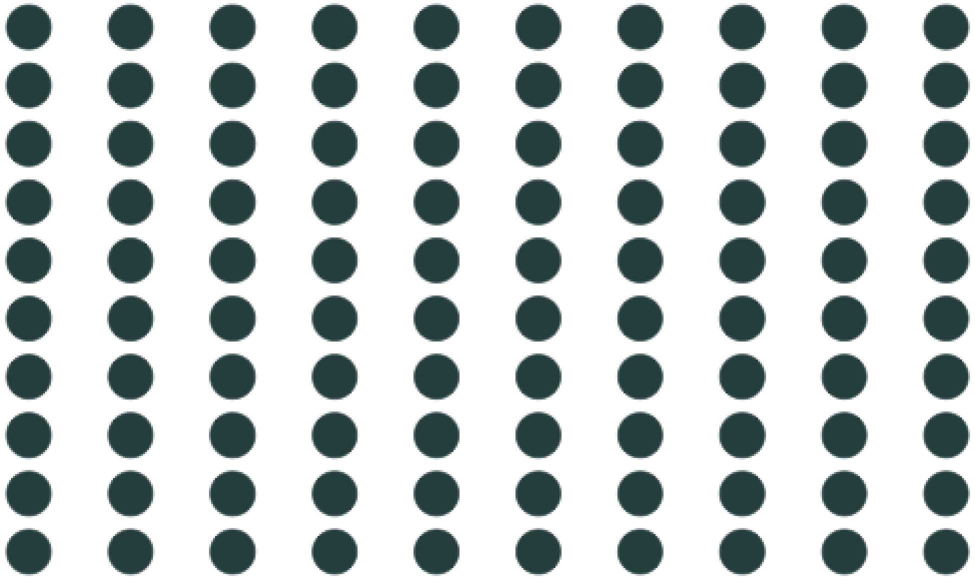
- OLS estimators ($\hat{\beta}_0$ and $\hat{\beta}_1$) are computed from a finite (specific) sample of data
- Our OLS model contains **2 sources of randomness**:
- ***Modeled* randomness**: u includes all factors affecting Y *other* than X
  - different samples will have different values of those other factors (u_i)
- ***Sampling* randomness**: different samples will generate different OLS estimators
  - Thus, $\hat{\beta}_0, \hat{\beta}_1$ are *also* **random variables**, with their own **sampling distribution**

The Two Problems: Where We're Heading...Ultimately

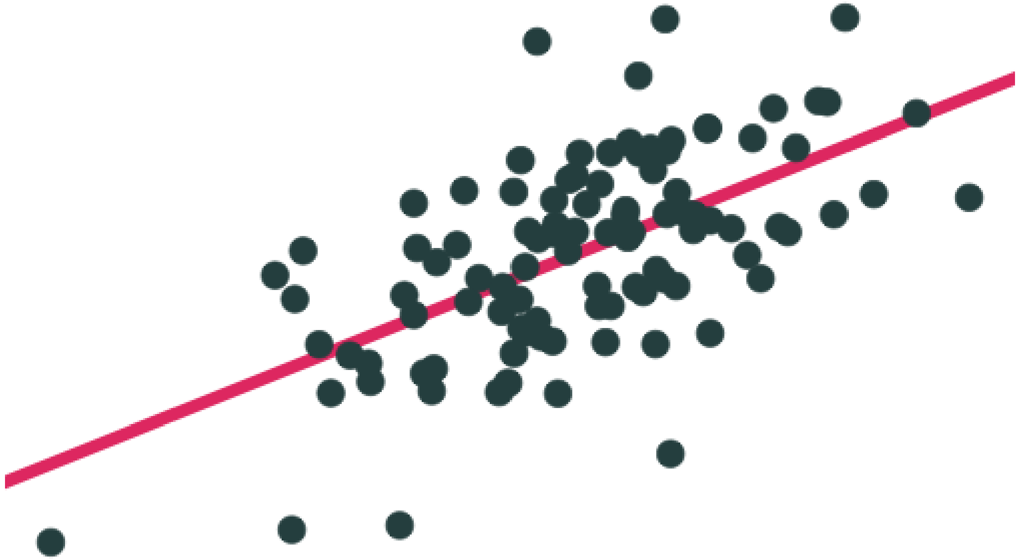
Sample $\xrightarrow{\text{statistical inference}}$ **Population** $\xrightarrow{\text{causal indentification}}$ **Unobserved Parameters**

- We want to **identify** causal relationships between **population** variables
  - Logically first thing to consider
  - **Endogeneity problem**
- We'll use **sample** *statistics* to **infer** something about population *parameters*
  - In practice, we'll only ever have a finite *sample distribution* of data
  - We *don't* know the *population distribution* of data
  - **Randomness problem**

Why Sample vs. Population Matters



Population

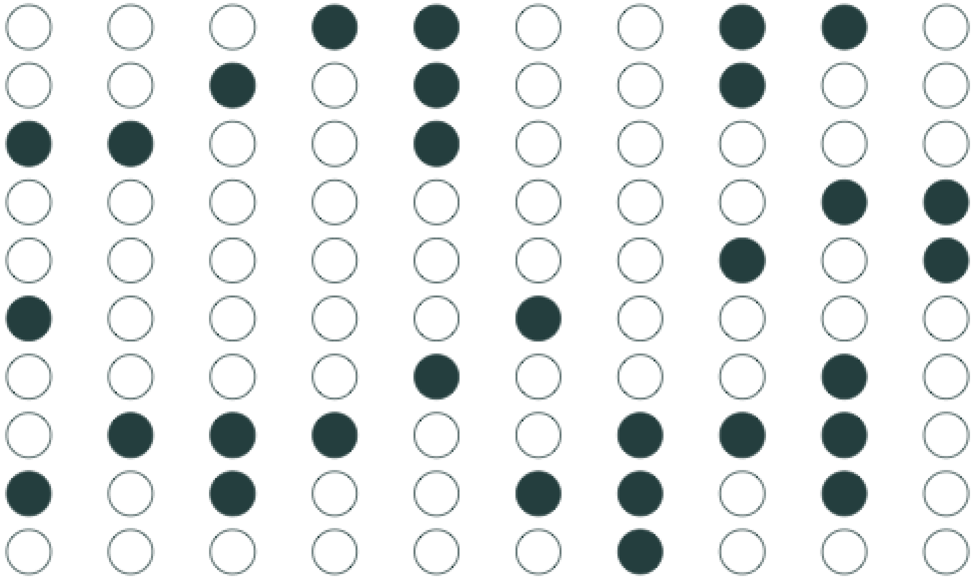


Population relationship

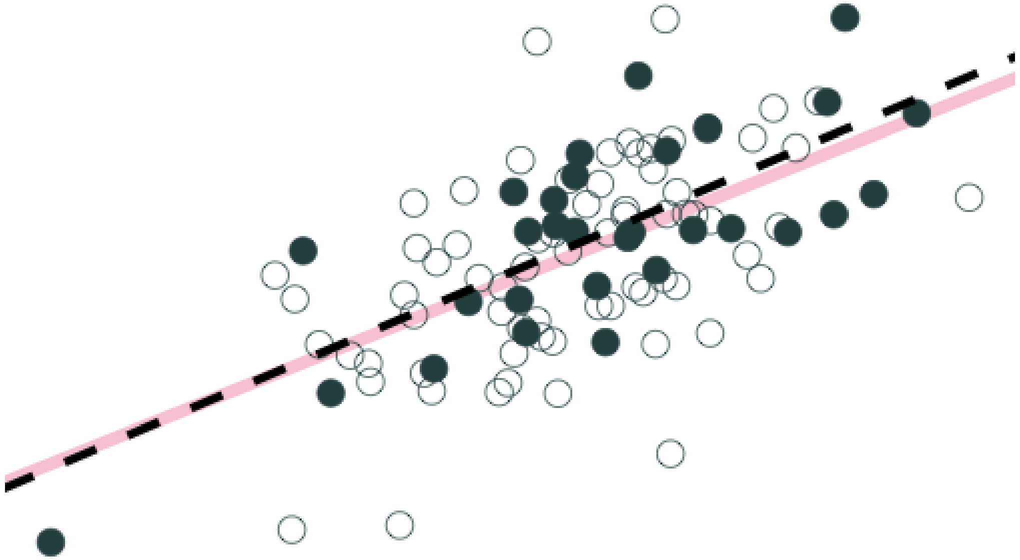
$$Y_i = 3.24 + 0.44X_i + u_i$$

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

Why Sample vs. Population Matters



Sample 1: 30 random individuals



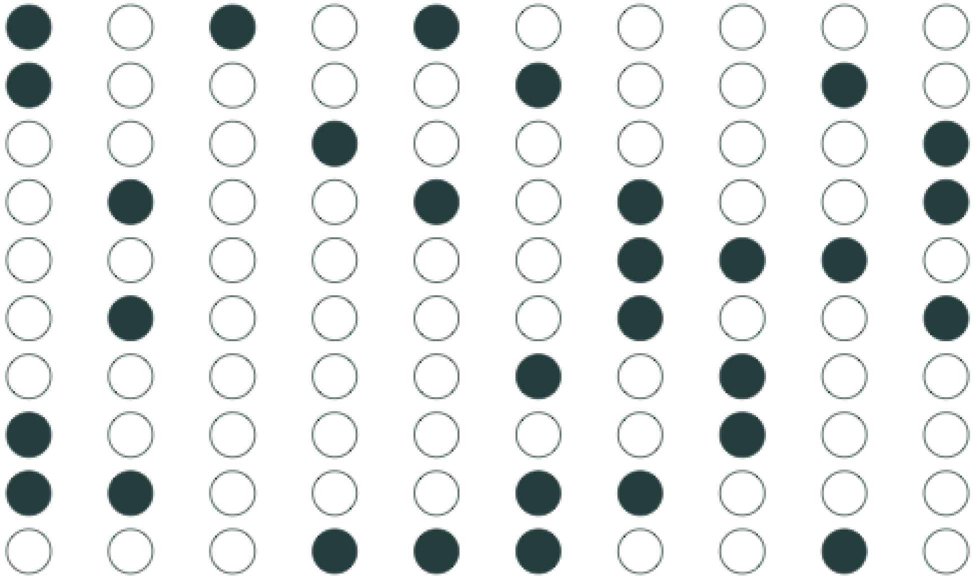
Population relationship

$$Y_i = 3.24 + 0.44X_i + u_i$$

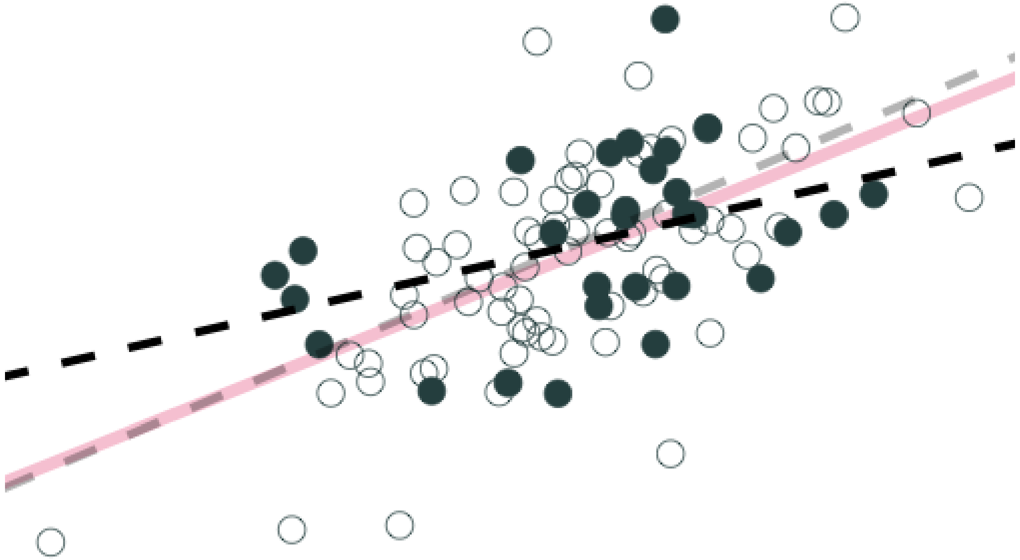
Sample relationship

$$\hat{Y}_i = 3.19 + 0.47X_i$$

Why Sample vs. Population Matters



Sample 2: 30 random individuals



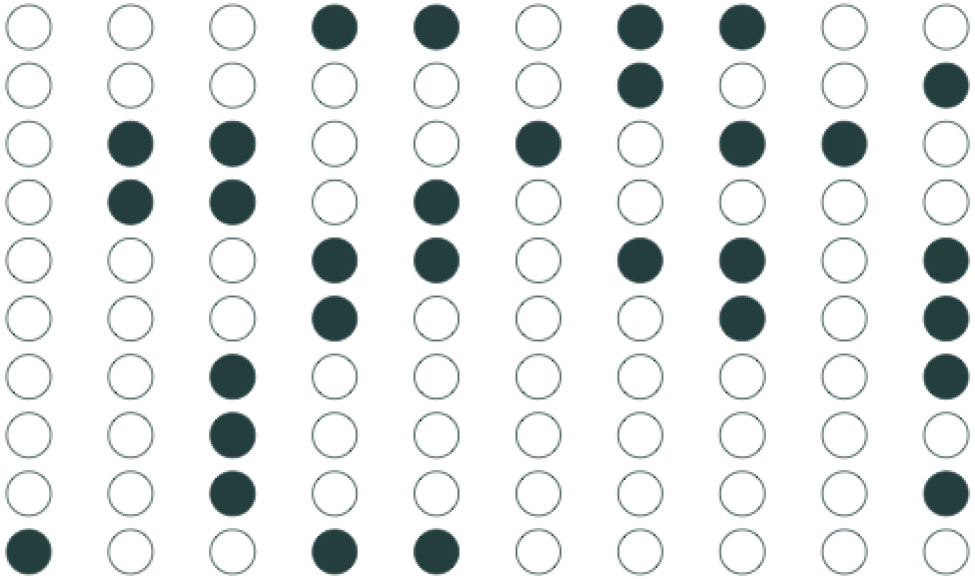
Population relationship

$$Y_i = 3.24 + 0.44X_i + u_i$$

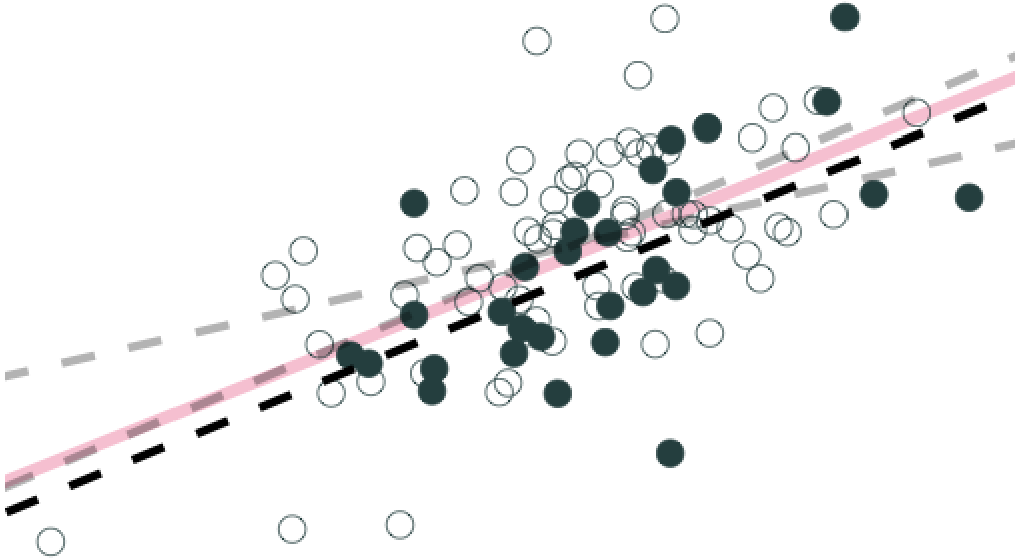
Sample relationship

$$\hat{Y}_i = 4.26 + 0.25X_i$$

Why Sample vs. Population Matters



Sample 3: 30 random individuals



Population relationship

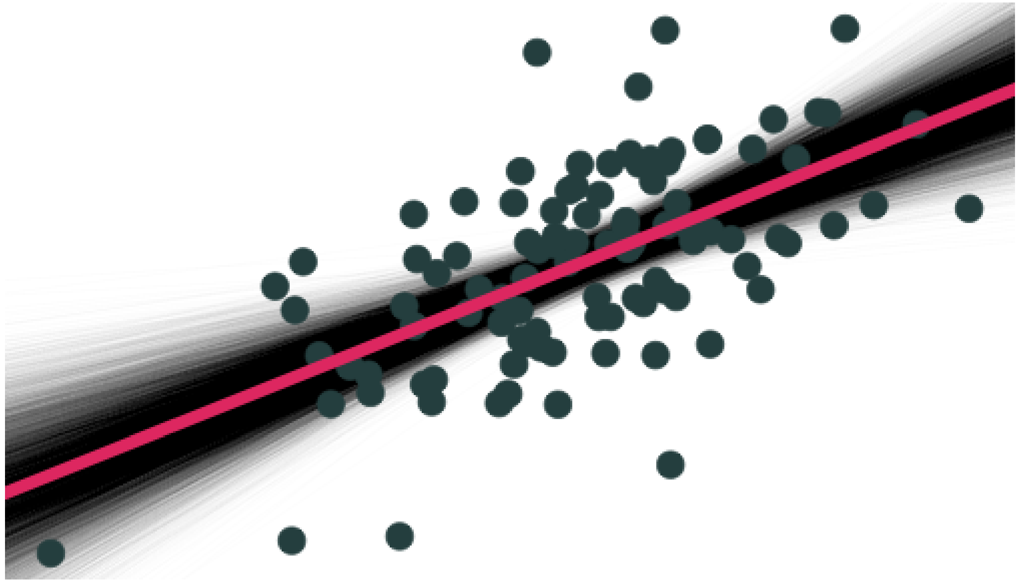
$$Y_i = 3.24 + 0.44X_i + u_i$$

Sample relationship

$$\hat{Y}_i = 2.91 + 0.46X_i$$

Why Sample vs. Population Matters

- Let's repeat this process **10,000 times**!
- This exercise is called a **(Monte Carlo) simulation**
  - I'll show you how to do this next class with the `infer` package



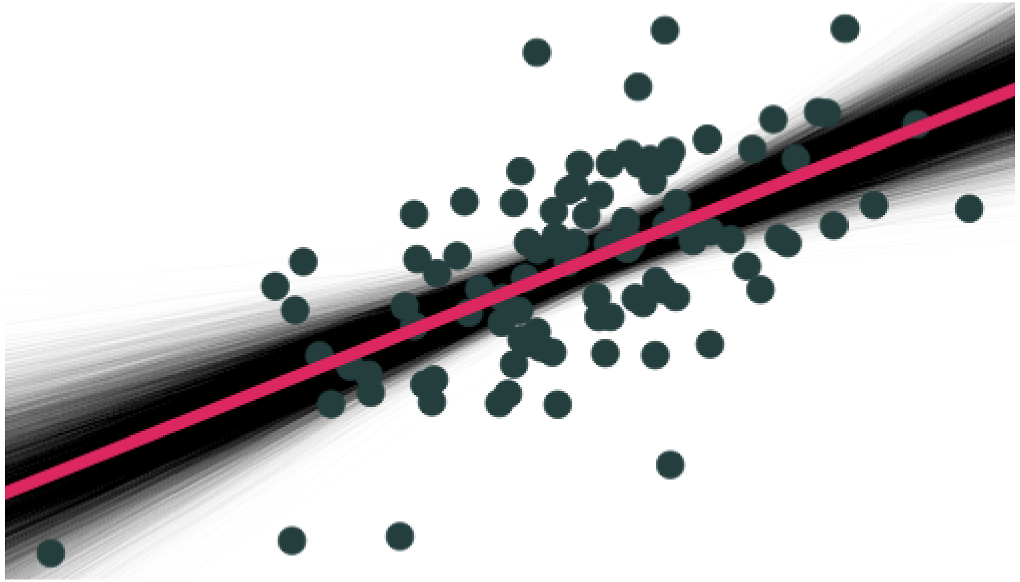
Why Sample vs. Population Matters

- **On average** estimated regression lines from our hypothetical samples provide an unbiased estimate of the true population regression line

$$E[\hat{\beta}_1] = \beta_1$$

- However, any *individual line* (any *one* sample) can miss the mark

- This leads to **uncertainty** about our estimated regression line
  - Remember, we only have *one* sample in reality!
  - This is why we care about the **standard error** of our line: $se(\hat{\beta}_1)$!



Confidence Intervals

Statistical Inference

$$\textbf{Sample} \xrightarrow{\text{statistical inference}} \textbf{Population} \xrightarrow{\text{causal indentification}} \textbf{Unobserved Parameters}$$

Statistical Inference

$$\textbf{Sample} \xrightarrow{\text{statistical inference}} \textbf{Population} \xrightarrow{\text{causal indentification}} \textbf{Unobserved Parameters}$$

- We want to start **inferring** what the true population regression model is, using our estimated regression model from our sample

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X \xrightarrow{\text{🤞 hopefully 🤞}} Y_i = \beta_0 + \beta_1 X + u_i$$

- We can't yet make **causal inferences** about whether/how X *causes* Y
  - coming after the midterm!

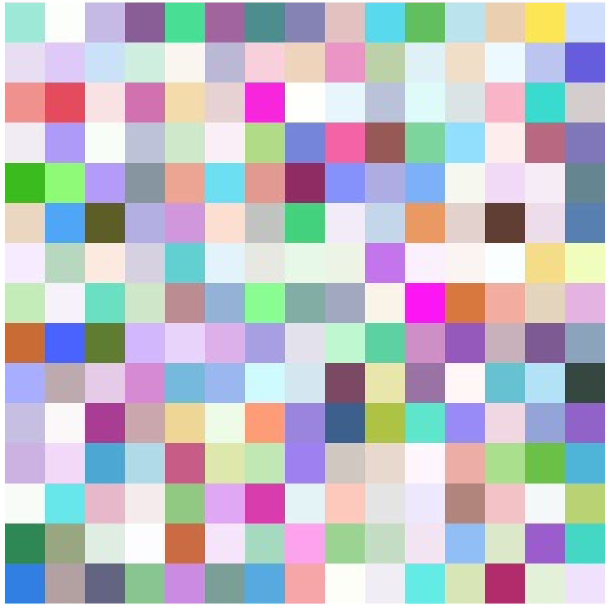
Estimation and Statistical Inference

- Our problem with **uncertainty** is we don't know whether our sample estimate is *close* or *far* from the unknown population parameter

- But we can use our errors to learn how well our model statistics likely estimate the true parameters

- Use $\hat{\beta}_1$ and its standard error, $se(\hat{\beta}_1)$ for statistical inference about true β_1

- We have two options...



Estimation and Statistical Inference

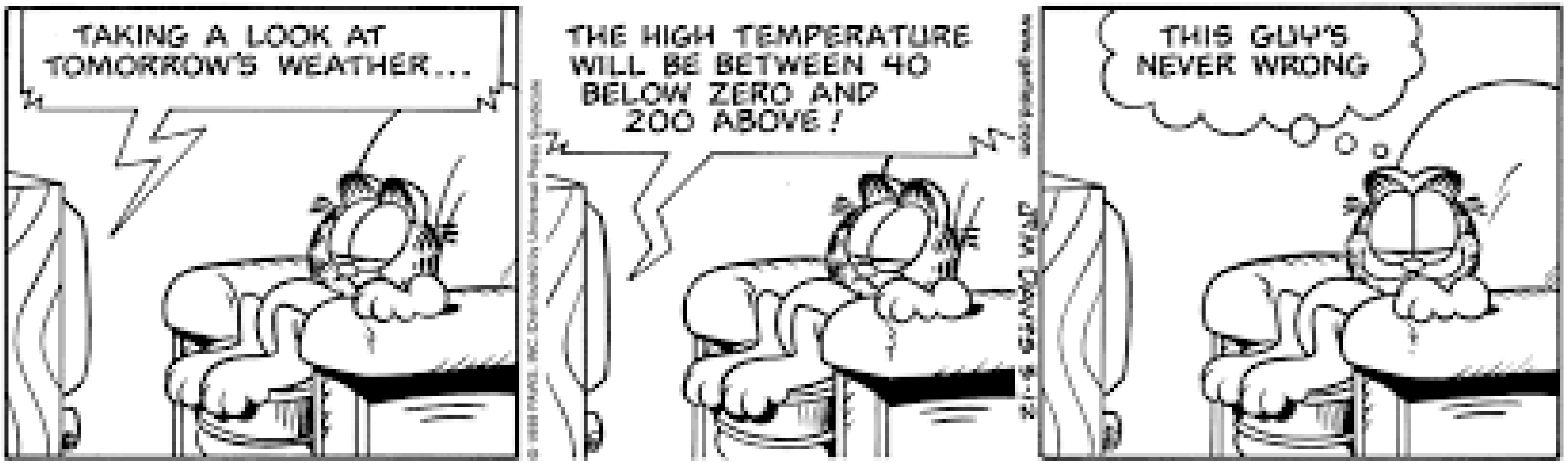
Point estimate

- Use our $\hat{\beta_1}$ and $se(\hat{\beta_1})$ to determine if we have statistically significant evidence to reject a hypothesized β_1

Confidence interval

- Use $\hat{\beta_1}$ and $se(\hat{\beta_1})$ to create an *range* of values that gives us a good chance of capturing the true β_1

Accuracy vs. Precision



- More typical in econometrics to do hypothesis testing (next class)

Generating Confidence Intervals

- We can generate our confidence interval by generating a **"bootstrap"** sampling distribution

- This takes our sample data, and resamples it by selecting random observations with replacement

- This allows us to approximate the sampling distribution of $\hat{\beta}_1$ by simulation!

Confidence Intervals Using the infer Package

Confidence Intervals Using the infer Package

- The `infer` package allows you to do statistical inference in a `tidy` way, following the philosophy of the `tidyverse`

```
# install first!
install.packages("infer")

# load
library(infer)
```

Confidence Intervals with the infer Package I

- `infer` allows you to run through these steps manually to understand the process:

1. `specify()` a model
2. `generate()` a bootstrap distribution
3. `calculate()` the confidence interval
4. `visualize()` with a histogram (optional)

Confidence Intervals with the infer Package II

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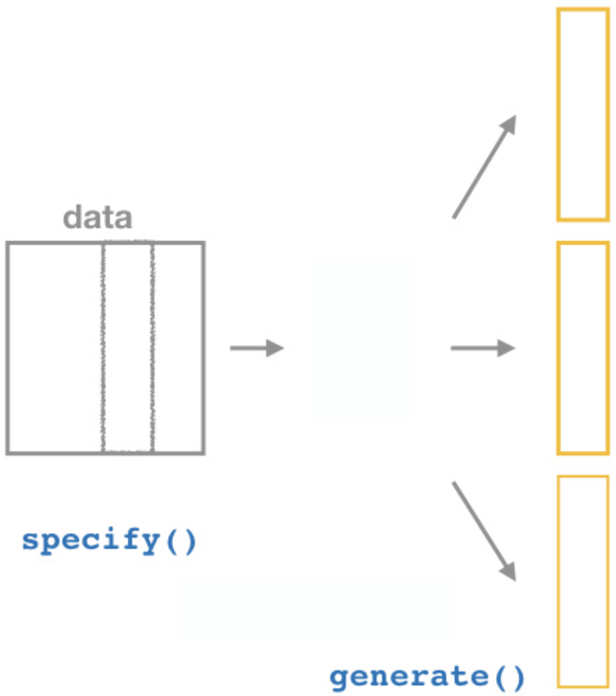
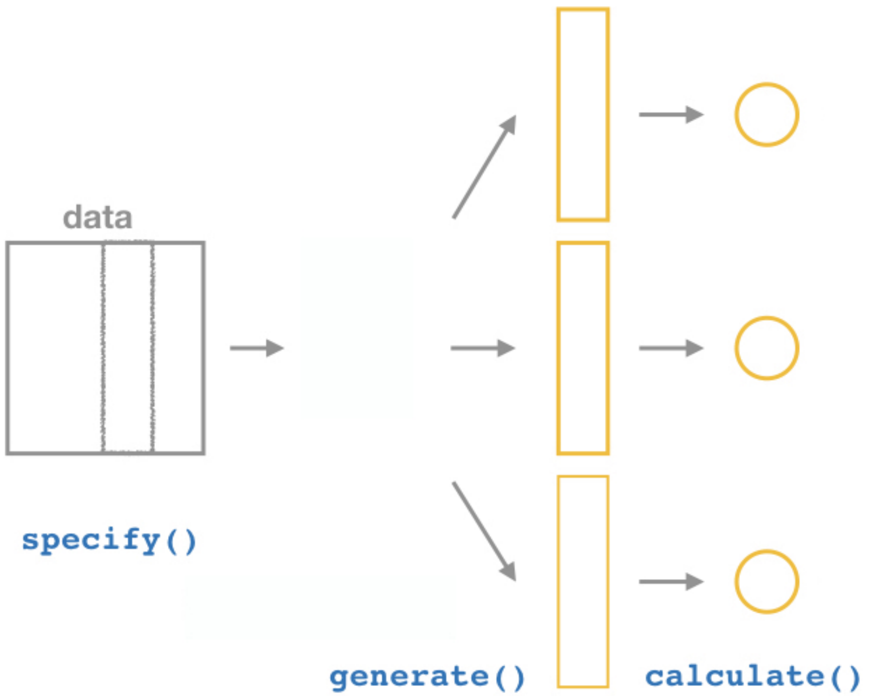
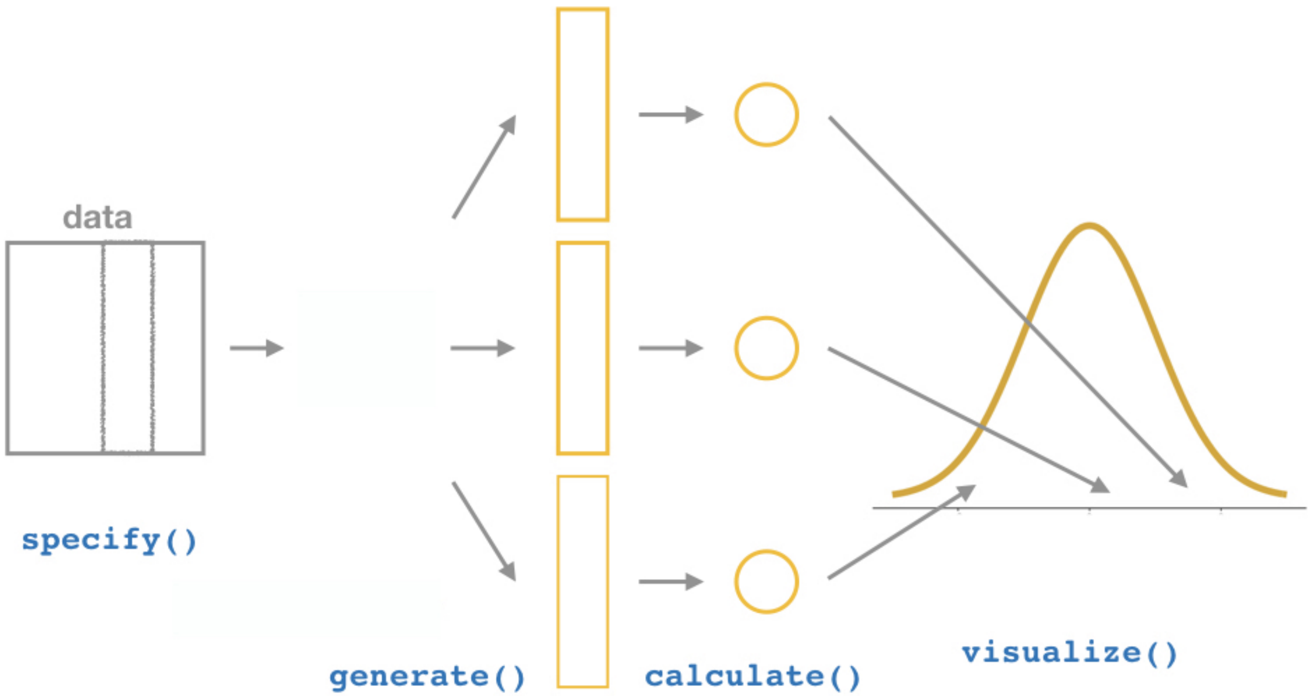


Confidence Intervals with the infer Package II



Confidence Intervals with the infer Package II



Bootstrapping

Our Sample

term	estimate	std.error	statistic
<chr>	<dbl>	<dbl>	<dbl>
(Intercept)	698.932952	9.4674914	73.824514
str	-2.279808	0.4798256	-4.751327

2 rows | 1-4 of 5 columns

Another "Sample"

term	estimate	std.error	statistic
<chr>	<dbl>	<dbl>	<dbl>
(Intercept)	708.270835	9.5041448	74.522311
str	-2.797334	0.4802065	-5.825274

2 rows | 1-4 of 5 columns

👆 Bootstrapped from Our Sample

- Now we want to do this 1,000 times to simulate the unknown sampling distribution of $\hat{\beta}_1$

The *infer* Pipeline: Specify

`specify()`

The *infer* Pipeline: Specify

Specify

`data %>% specify(y ~ x)`

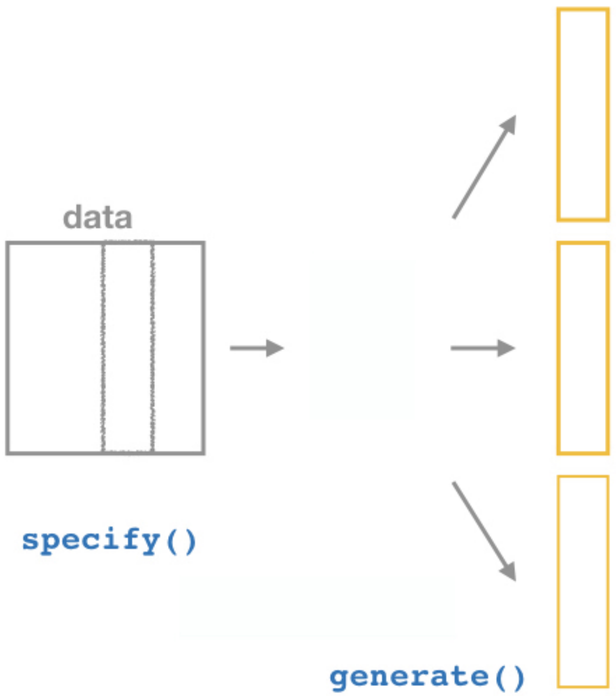
- Take our data and pipe it into the `specify()` function, which is essentially a `lm()` function for regression (for our purposes)

```
CASchool %>%
  specify(testscr ~ str)
```

testscr <dbl>	str <dbl>
690.80	17.88991
661.20	21.52466
643.60	18.69723
647.70	17.35714
640.85	18.67133

5 rows

The *infer* Pipeline: Generate



The *infer* Pipeline: Generate

Specify

Generate

`%>% generate(reps = n, type = "bootstrap")`

- Now the magic starts, as we run a number of simulated samples
- Set the number of `reps` and set `type` to `"bootstrap"`

```
CASchool %>%
  specify(testscr ~ str) %>%
  generate(reps = 1000,
           type = "bootstrap")
```

The *infer* Pipeline: Generate

Specify

Generate

```
%>% generate(reps = n,
type = "bootstrap")
```

replicate	testscr	str
<int>	<dbl>	<dbl>
1	642.20	19.22221
1	664.15	19.93548
1	671.60	20.34927
1	640.90	19.59016
1	677.25	19.34853
1	672.20	20.20000
1	621.40	22.61905
1	657.00	20.86808
1	664.95	25.80000
1	635.20	17.75499

1-10 of 10,000 rows Previous **1** 2 3 4 5 6 ... 1000 Next

- `replicate`: the "sample" number (1-1000)
- creates `x` and `y` values (data points)

The *infer* Pipeline: Calculate

Specify

Generate

Calculate

`%>% calculate(stat = "slope")`

```
CASchool %>%
  specify(testscr ~ str) %>%
  generate(reps = 1000,
           type = "bootstrap") %>%
  calculate(stat = "slope")
```

- For each of the 1,000 replicates, calculate `slope` in `lm(testscr ~ str)`
- Calls it the `stat`

The *infer* Pipeline: Calculate

Specify

Generate

Calculate

```
%>% calculate(stat = "slope")
```

replicate <int>	stat <dbl>
1	-3.0370939
2	-2.2228021
3	-2.6601745
4	-3.5696240
5	-2.0007488
6	-2.0979764
7	-1.9015875
8	-2.5362338
9	-2.3061820
10	-1.9369460

1-10 of 1,000 rows Previous **1** 2 3 4 5 6 ... 100 Next

The *infer* Pipeline: Calculate

Specify

Generate

Calculate

`%>% calculate(stat = "slope")`

```r
boot <- CASchool %>% #<< # save this
  specify(testscr ~ str) %>%
  generate(reps = 1000,
           type = "bootstrap") %>%
  calculate(stat = "slope")
```

- `boot` is (our simulated) sampling distribution of $\hat{\beta}_1$!
- We can now use this to estimate the confidence interval from *our* $\hat{\beta}_1 = -2.28$
- And visualize it

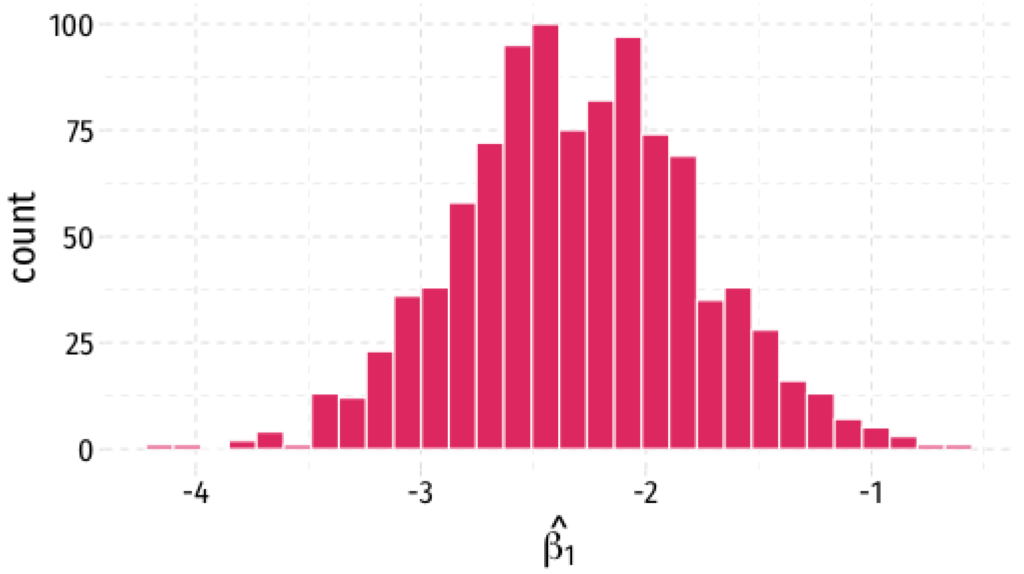
Confidence Interval

- A 95% confidence interval is the middle 95% of the sampling distribution

lower	upper
<dbl>	<dbl>
-3.340545	-1.238815

1 row

```
sampling_dist<-ggplot(data = boot)+
  aes(x = stat)+
  geom_histogram(color="white", fill = "#e64173
  labs(x = expression(hat(beta[1])))+
  theme_pander(base_family = "Fira Sans Condens
           base_size=20)
sampling_dist
```



Confidence Interval

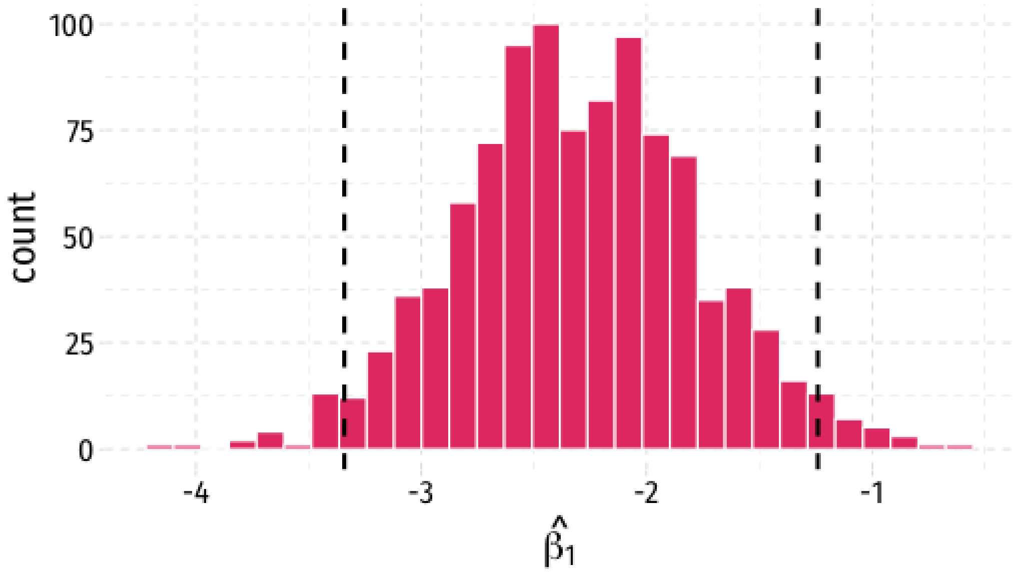
- A confidence interval is the middle 95% of the sampling distribution

```r
ci<-boot %>%
  summarize(lower = quantile(stat, 0.025),
            upper = quantile(stat, 0.975))
ci
```

	lower	upper
	<dbl>	<dbl>
	-3.340545	-1.238815

1 row

```r
sampling_dist+
  geom_vline(data = ci, aes(xintercept = lower))+
  geom_vline(data = ci, aes(xintercept = upper))
```



The *infer* Pipeline: Confidence Interval

Specify

Generate

Calculate

Get Confidence Interval

`%>%` `get_confidence_interval()`

```r
CASchool %>% #<< # save this
  specify(testscr ~ str) %>%
  generate(reps = 1000,
           type = "bootstrap") %>%
  calculate(stat = "slope") %>%
  get_confidence_interval(level = 0.95,
                          type = "se",
                          point_estimate = -2.28)
```

lower_ci <dbl>	upper_ci <dbl>
-3.273376	-1.286624

1 row

Broom Can Estimate a Confidence Interval

```r
tidy_reg <- school_reg %>% tidy(conf.int = T)
tidy_reg
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
(Intercept)	698.932952	9.4674914	73.824514	6.569925e-242	680.32313	717.542779
str	-2.279808	0.4798256	-4.751327	2.783307e-06	-3.22298	-1.336637

2 rows

```r
# save and extract confidence interval
our_CI <- tidy_reg %>%
  filter(term == "str") %>%
  select(conf.low, conf.high)

our_CI
```

conf.low	conf.high
<dbl>	<dbl>
-3.22298	-1.336637

1 row

The *infer* Pipeline: Confidence Interval

Specify

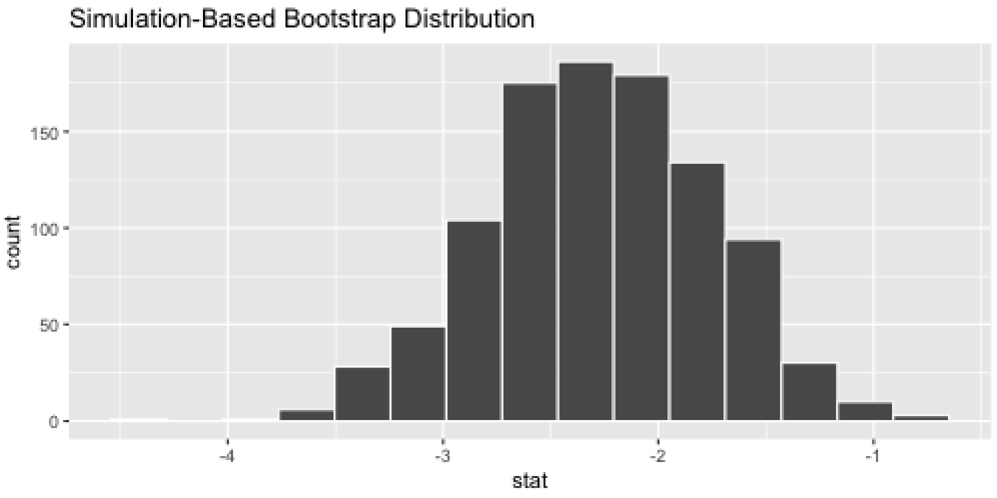
Generate

Calculate

Visualize

`%>% visualize()`

```
CASchool %>% #<< # save this
  specify(testscr ~ str) %>%
  generate(reps = 1000,
           type = "bootstrap") %>%
  calculate(stat = "slope") %>%
  visualize()
```



- `visualize()` is just a wrapper for `ggplot()`

The *infer* Pipeline: Confidence Interval

Specify

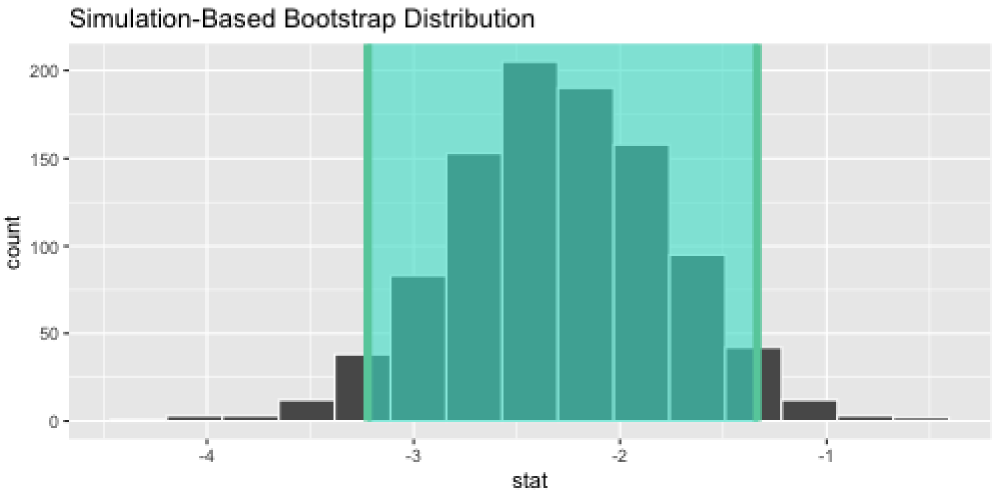
Generate

Calculate

Visualize

`%>% visualize()`

```
CASchool %>% #<< # save this
  specify(testscr ~ str) %>%
  generate(reps = 1000,
           type = "bootstrap") %>%
  calculate(stat = "slope") %>%
  visualize()+shade_ci(endpoints = our_CI)
```



- If we have our confidence levels saved (`our_CI`) we can `shade_ci()` in `infer`'s `visualize()` function

Confidence Intervals

- In general, a **confidence interval (CI)** takes a point estimate and extrapolates it within some **margin of error (MOE)**:

$$\left(\left[\text{estimate - MOE}\right], \left[\text{estimate + MOE}\right]\right)$$

- The main question is, **how confident do we want to be** that our interval contains the true parameter?
  - Larger confidence level, larger margin of error (and thus larger interval)

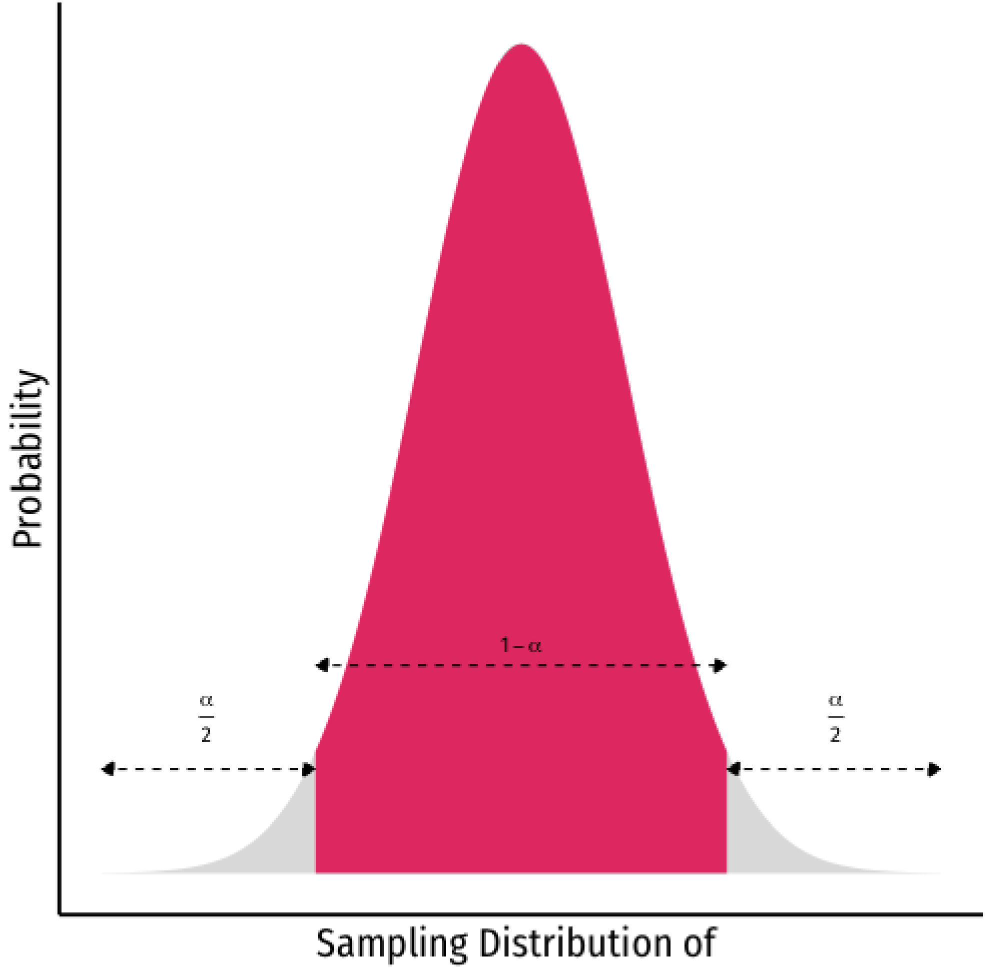
Confidence Intervals

- $(1-\alpha)$ is the **confidence level** of our confidence interval
  - α is the **"significance level"** that we use in hypothesis testing
  - $\alpha =$ probability that the true parameter is *not* contained within our interval
- Typical levels: 90%, 95%, 99%
  - 95% is especially common, $\alpha = 0.05$

Confidence Levels

- Depending on our confidence level, we are essentially looking for the middle $(1-\alpha)\%$ of the sampling distribution

- This puts α in the tails; $\frac{\alpha}{2}$ in each tail

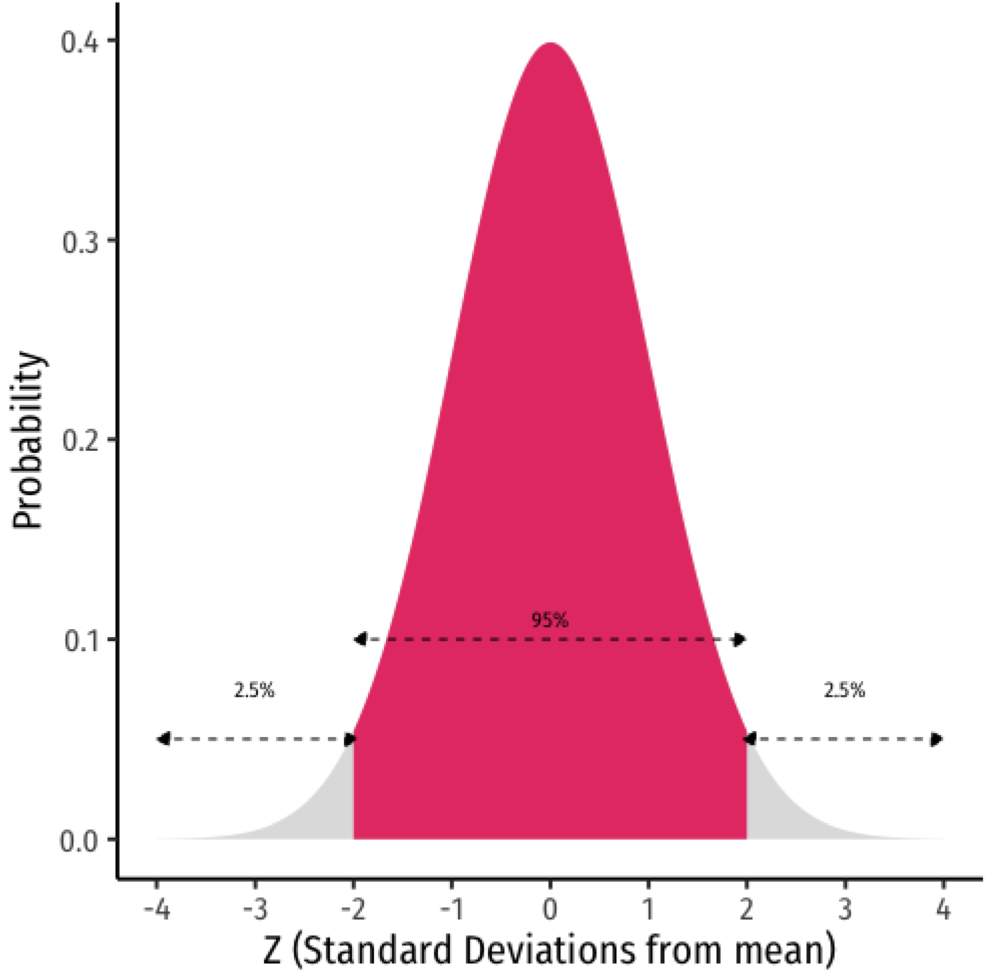


Confidence Levels and the Empirical Rule

- Recall the **68-95-99.7% empirical rule** for (standard) normal distributions![†]

- 95% of data falls within 2 standard deviations of the mean

- Thus, in 95% of samples, the true parameter is likely to fall within *about* 2 standard deviations of the sample estimate



[†] I'm playing fast and loose here, we can't actually use the normal distribution, we use the Student's t-distribution with n-k-1 degrees of freedom. But there's no need to complicate things you don't need to know about. Look at today's class notes for more.

Interpreting Confidence Intervals

- So our confidence interval for our slope is (-3.22, -1.33), what does this mean again?

❌ 95% of the time, the true effect of class size on test score will be between -3.22 and -1.33

❌ We are 95% confident that a randomly selected school district will have an effect of class size on test score between -3.22 and -1.33

❌ The effect of class size on test score is -2.28 95% of the time.

✅ We are 95% confident that in similarly constructed samples, the true effect is between -3.22 and -1.33

Estimating in R (Via Regression, rather than `infer`)

- Base R doesn't show confidence intervals in the `lm summary()` output, need the `confint` command

```
confint(school_reg)
```

```
##                  2.5 %     97.5 %
## (Intercept) 680.32313 717.542779
## str          -3.22298  -1.336637
```

- `broom` can include confidence intervals

```
school_reg %>%
  tidy(conf.int = TRUE)
```

term	estimate	std.error
<chr>	<dbl>	<dbl>
(Intercept)	698.932952	9.4674914
str	-2.279808	0.4798256

2 rows | 1-3 of 7 columns